(Part II: Security of Economics)

Lecture 5: Market with intermediaries and advertising

Dusko Pavlovic

Spring 2013

Outline

Outline

Introduction

Market is a system of exchange protocols

- compute the prices
- regulate the exchange

Introduction

Market is a system of exchange protocols

- compute the prices
- regulate the exchange

We focus on computing the prices.

Introduction

An auction is a market organized by

- a seller: supply auction
- a buyer: procurement auction

Introduction

Markets in general are organized by

- **universal buyers/sellers**
  - merchants, traders, dealers,
  - entrepreneurs,
  - advertisers (push), solicitors (pull)

  who mediate among the buyers and the sellers

Introduction

II-5. Intermediaries
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Markets in general are organized by

- **universal buyers/sellers**
  - merchants, traders, dealers,
  - entrepreneurs,
  - advertisers (push), solicitors (pull)

  who mediate among the buyers and the sellers

- just like the **universal goods**
  - money
  - securities (bonds, equity, derivatives)

  mediate among the goods

In this lecture

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- Multi-item auctions
  - example: sponsored search
  - problem of incentive compatibility
    - Later: *What is the value of advertising?*
- Market with intermediaries
  - traders' strategies
  - trading profits and social benefits

Outline

Sponsored search setting

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clickthrough rates	slots	advertisers	revenues per click
10	a	x	3
5	b	y	2
2	c	z	1

Sponsored search as a matching problem

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slots	advertisers	valuations
a	x	30, 15, 6
b	y	20, 10, 4
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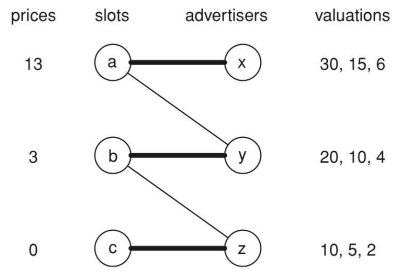
Sponsored search as a market



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Market mechanism

- n buyers, n item
  - take $n = \{0, 1, \ldots, n-1\}$

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- matching $\sigma_{vp} : n \to n$ assigns item $\sigma_{vp}(i)$ to i
- i's utility $u_i \in \mathbb{R}$ is

$$u_i = v_{i\sigma_{vp}(i)} - p_{\sigma_{vp}(i)}$$

Goal of the market mechanism

Maximize social welfare, i.e. buyers' total payoff

$$\begin{aligned} U(v,p) &= \sum_{i\in n} u_i \\ &= \sum_{i\in n} v_{i\sigma_{vp}(i)} - p_{\sigma_{vp}(i)} \\ &= \sum_{i\in n} v_{i\sigma(i,v)} - P \end{aligned}$$

where $P = \sum_{i<n} p_i$

Markets respect preference

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To maximize utility, $\sigma_{vp} : n \to n$ maximizes valuations

$$v_{i\sigma(i,v)} \geq v_{ij}$$

Position auction mechanism

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- n bidders, n positions

Position auction mechanism

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- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
  - bidders' valuations per click $w = (w_i)_n$
  - position click-through rates $r = (r_j)_n$

Position auction mechanism

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- bidders bid $b = (b_i)_n$
- price per position $\pi_{ij}(b) = p_i(b) \cdot r_j$ where
  - price per click $p(b) = \big(p_i(b)\big)_n$

Position auction mechanism

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- matching $\tau : n \times \mathbb{R}^n \to n$ assigns item $\tau(i, b)$ to i

Position auction mechanism

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- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) \;=\; v_{i\tau(i,b)} - \pi_{i\tau(i,b)}(b) \;=\; \big(w_i - p_i(b)\big) \cdot r_{\tau(i,b)}$$

Goal of the position auction mechanism

Maximize seller's revenue

$$P(b) = \sum_{i<n} \pi_{i\tau(i,b)}(b) = \sum_{i<n} p_i(b) \cdot r_{\tau(i,b)}$$

where

- all p_i grow with b
- bidder i bids b_i to maximize $u_i(b)$.

Position auctions respect preference

To maximize $p_i(b)$ with $u_i(b)$ always use

- $\tau(i, b) < \tau(j, b) \implies b_i \geq b_j$, i.e.
- $\tau(i, b) = j$ if b_i is j-th largest entry in b

Assumption

- The bidders are ordered by their bids

$$b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n$$

- The positions are ordered by click-through rates

$$r_1 \geq r_2 \geq r_3 \geq \cdots \geq r_n$$

Generalized Second Price Auction

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
  - bidders' valuations per click $w = (w_i)_n$
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Generalized Second Price Auction

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- price per click $p_i(b) = b_{i+1}$

Generalized Second Price Auction

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- bidders bid $b = (b_i)_n$
- price per click $p_i(b) = b_{i+1}$
- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) = \left(w_i - b_{i+1}\right) \cdot r_i$$

Does GSPA encourage truthful bidding?

clickthrough rates	slots	advertisers	revenues per click
10	a	x	7
4	b	y	6
0	c	z	1

Does GSPA encourage truthful bidding?

clickthrough rates	slots	advertisers	revenues per click
10	a	x	7
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- with truthful bid: $u_x(7,6,1) = (7-6) \cdot 10 = 10$
- with untruthful bid: $u_x(5,6,1) = (7-1) \cdot 4 = 24$

Position auction example

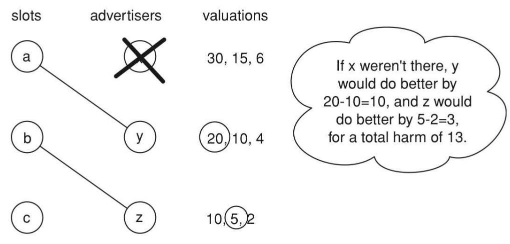
clickthrough rates	slots	advertisers	revenues per click
10	a	x	3
5	b	y	2
2	c	z	1

Matching problem view

slots	advertisers	valuations
a	x	30, 15, 6
b	y	20, 10, 4
c	z	10, 5, 2

Idea

- How much does x subtract from social welfare?



Idea

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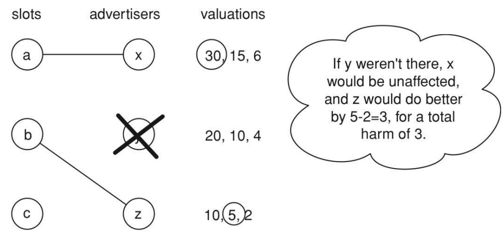
- How much does y subtract from social welfare?



Idea: Vickrey, Clarke, Groves

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- Each bidder should pay the cost that their bid incurs on social welfare
  - i.e., the sum of the losses that they cause to other bidders

Vickrey-Clarke-Groves Auction

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Notation

- B — set of bidders
- S — set of sellers (items)
- $v = (v_{ij})_{B\times S}$ — bidders' valuations
- V_B^S — maximal total valuation

Vickrey-Clarke-Groves Auction

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Notation

- B — set of bidders
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- V_B^S — maximal total valuation

Remark

- If $\#B < \#S$, then add $\#S - \#B$ bidders with all valuations 0
- If $\#B > \#S$, then add $\#B - \#S$ sellers valued 0 by all.

Vickrey-Clarke-Groves Auction

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Remember the assumption

- The bidders are ordered by their bids

$$b_1 \geq b_2 \geq b_3 \geq \cdots \geq b_n$$

- The positions are ordered by click-through rates

$$r_1 \geq r_2 \geq r_3 \geq \cdots \geq r_n$$

Vickrey-Clarke-Groves Auction

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
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- bidders bid $b = (b_i)_n$
- price per item $\pi_{ij}(b) = V^S_{B\setminus i} - V^{S\setminus j}_{B\setminus i}$

Vickrey-Clarke-Groves Auction

- n bidders, n positions
- bidders' valuations $v_{ij} = w_i \cdot r_j$ where
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- price per item $\pi_{ij}(b) = V^S_{B\setminus i} - V^{S\setminus j}_{B\setminus i}$
- i's utility $u_i : \mathbb{R}^n \to \mathbb{R}$ is

$$u_i(b) = v_{ii} - \pi_{ii}(b)$$

Vickrey-Clarke-Groves Auction

Theorem

The VCG auction is incentive compatible: truthful bidding is the unique Nash equilibrium for all players.

Vickrey-Clarke-Groves Auction

Corollary

The VCG auction maximizes social wellfare, i.e. the total utility of bidders.

Problem

Homework

For the sponsored search market

clickthrough rates	slots	advertisers	revenues per click
10	a	x	7
4	b	y	6
0	c	z	1

compute seller's revenue (i.e. the total of the prices charged for all items) if the positions are auctioned by a GSP auction and by a VCG auction

Show that neither of these mechanisms maximizes seller's revenue.

Billion \$ problem

Design an auction mechanism that maximizes seller's revenue.

Outline

Toy market

- There is just one type of goods.
- Every buyer needs to buy one item.
- Every seller needs to sell one item.

Toy market

- buyers $\mathcal{B} = \{B_1, B_2, \ldots B_n\}$ have valuations v_i
- sellers $\mathcal{S} = \{S_1, S_2, \ldots S_n\}$ have valuations w_j

Toy market

- buyers $\mathcal{B} = \{B_1, B_2, \ldots B_n\}$ have valuations v_i
- sellers $\mathcal{S} = \{S_1, S_2, \ldots S_n\}$ have valuations w_j

Remark

If the numbers are different, then add

- buyers with the valuation 0, or
- sellers with the valuation 1.

Toy market

Goal of the market

Find a bijection $\sigma : \mathcal{B} \to \mathcal{S}$ that maximizes social benefit

$$SB_\sigma = \sum_{i=1}^{n} v_i - w_{\sigma i}$$

Market with intermediaries

- Just like the goods are compared through **universal goods**
  - money, securities
- the buyers' and the sellers' are connected through **universal buyers/sellers**
  - merchants, traders, advertisers

Market with intermediaries

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The intermediaries mediate the flows

- merchants buy, move and sell goods
- traders buy and sell goods without moving them
- advertisers and solicitors move information

Market with intermediaries

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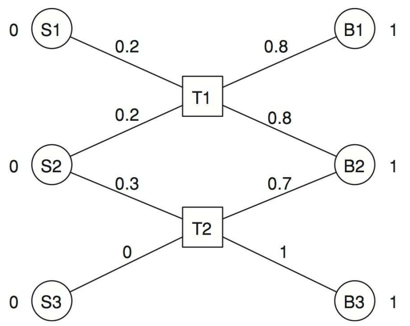

Market with intermediaries as a game

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- buyers $\mathcal{B} = \{B_1, B_2, B_3\}$
  - their reserve prices (valuations) $v_1 = v_2 = v_3 = 1$
- sellers $\mathcal{S} = \{S_1, S_2, S_3\}$
  - their reserve price (valuations) $w_1 = w_2 = w_3 = 0$
- traders $\mathcal{T} = \{T_1, T_2\}$
  - ask relation $T_1 \xrightarrow{a} B_1$, $T_1 \xrightarrow{a} B_2$, $T_2 \xrightarrow{a} B_2$, $T_2 \xrightarrow{a} B_3$
    - T_1's buyers $\mathcal{B}_1 = \{B_1, B_2\}$
    - T_2's buyers $\mathcal{B}_2 = \{B_2, B_3\}$
  - bid relation $S_1 \xrightarrow{b} T_1$, $S_2 \xrightarrow{b} T1$, $S_2 \xrightarrow{b} T_2$, $S_3 \xrightarrow{b} T_2$
    - T_1's sellers $\mathcal{S}_1 = \{S_1, S_2\}$
    - T_2's sellers $\mathcal{S}_2 = \{S_2, S_3\}$

Market with intermediaries as a game

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Setting

- buyers $\mathcal{B} = \{B_1, \ldots, B_n\}$
  - B_i's reserve price (valuation) is v_i
- sellers $\mathcal{S} = \{S_1, \ldots, S_n\}$
  - S_j's reserve price (valuation) is w_j
- traders $\mathcal{T} = \{T_1, \ldots T_m\}$
  - ask relation $\xrightarrow{a} \subseteq \mathcal{T} \times \mathcal{B}$
    - T_k's buyers $\mathcal{B}_k = \{B_i \in \mathcal{B} \mid T_k \xrightarrow{a} B_i\}$
  - bid relation $\xrightarrow{b} \subseteq \mathcal{S} \times \mathcal{T}$
    - T_k's sellers $\mathcal{S}_k = \{S_j \in \mathcal{S} \mid S_j \xrightarrow{b} T_k\}$

Market with intermediaries as a game

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Game

players: traders $T_1, \ldots, T_m$

moves: for the trader T_k's the set of moves is

$$\begin{aligned} P_k &= Pb_k \times Pa_k, \quad \text{where} \\ Pb_k &= \mathbb{R}^p \text{ with } p = \#\mathcal{S}_k \\ Pa_k &= \mathbb{R}^q \text{ with } q = \#\mathcal{B}_k \end{aligned}$$

where

- $b_k = \langle b_{k1}, b_{k2}, \ldots, b_{kp} \rangle \in Pb_k$ are T_k's bid prices for all $S_j \in \mathcal{S}_k$
- $a_k = \langle a_{k1}, a_{k2}, \ldots, a_{kq} \rangle \in Pa_k$ are T_k's ask prices for all $B_i \in \mathcal{B}_k$

Market with intermediaries as a game

Play

- Each T_k announces its bid and ask prices $p_k = \langle b_k, a_k \rangle$
- Each S_j agrees to sell to a T_k with a maximal b_{kj}
- Each B_i agrees to buy from a T_k with a minimal a_{ki}
- Each T_k thus forms the sets of
  - suppliers $\mathcal{MS}_k = \left\{S_j \in \mathcal{S}_k \mid \forall \ell.\ b_{\ell j} \le b_{kj}\right\}$
  - customers $\mathcal{MB}_k = \{B_i \in \mathcal{B}_k \mid \forall \ell.\ a_{ki} \le a_{\ell i}\}$

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Market with intermediaries as a game

Trader T_k's utility

- If $\#\mathcal{MB}_k \le \#\mathcal{MS}_k$ (sufficient supplies) then

$$u_k(\vec{p}) = \sum_{B_i \in \mathcal{MB}_k} a_{ki} - \sum_{S_j \in \mathcal{MS}_k} b_{kj}$$

- If $\#\mathcal{MB}_k > \#\mathcal{MS}_k$ (insufficient supplies) then

$$u_k(\vec{p}) = \sum_{B_i \in \mathcal{MB}_k^+} a_{ki} - \sum_{S_j \in \mathcal{MS}_k} b_{kj} - \sum_{B_i \in \mathcal{MB}_k^-} a_{ki}$$

  where $\mathcal{MB}_k = \mathcal{MB}_k^+ \cup \mathcal{MB}_k^-$, and
  - $\mathcal{MB}_k^+$ is the set of $\#\mathcal{MS}_k$ buyers who accepted the highest ask prices
  - $\mathcal{MB}_k^-$ are the remaining $\#\mathcal{MB}_k - \#\mathcal{MS}_k$ buyers with the lowest ask prices

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Distribution of social benefit

If the bijection $\sigma : \mathcal{B} \to \mathcal{S}$ that maximizes social benefit

$$SB_\sigma = \sum_{i=1}^{n} v_i - w_{\sigma i}$$

is found through the traders $\kappa : \mathcal{B} \to \mathcal{T}$, then the benefit is distributed

$$SB_\sigma = \sum_{i=1}^{n} \underbrace{\left(v_i - a_{\kappa(i)i}\right)}_{UB} + \underbrace{\left(a_{\kappa(i)i} - b_{\kappa(i)\sigma(i)}\right)}_{UT} + \underbrace{\left(b_{\kappa(i)\sigma(i)} - w_{\sigma i}\right)}_{US}$$

where

- UB is the utility of the buyer
- UT is the utility of the trader
- US is the utility of the seller

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$$SB_\sigma = \sum_{i=1}^{n} \underbrace{\left(v_i - a_{\kappa(i)i}\right)}_{UB} + \underbrace{\left(a_{\kappa(i)i} - b_{\kappa(i)\sigma(i)}\right)}_{UT} + \underbrace{\left(b_{\kappa(i)\sigma(i)} - w_{\sigma i}\right)}_{US}$$

where

- UB is the utility of the buyer
- UT is the utility of the trader
- US is the utility of the seller

The traders maximize UT.

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Distribution of social benefit

- But how do the traders achieve their payoffs?
- What are the equilibria in the trading game?

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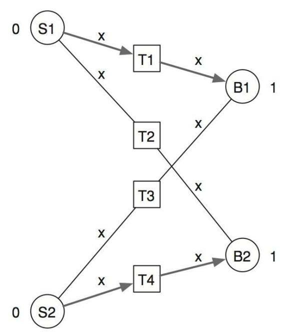
Implicit perfect competition



Indifference principle

At equilibrium

- All bid prices offered to a seller must be equal
- The seller will accept the bid from the trader who has access to the highest paying buyers
  - because that trader can increase the bid by ε

Indifference principle

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At equilibrium

- All bid prices offered to a seller must be equal
- The seller will accept the bid from the trader who has access to the highest paying buyers
  - because that trader can increase the bid by ε
- All ask prices offered to a buyer must be equal
- The buyer will accept the offer from the trader who has access to the lowest charging sellers
  - because that trader can undercut the offer by ε

Ripple effects

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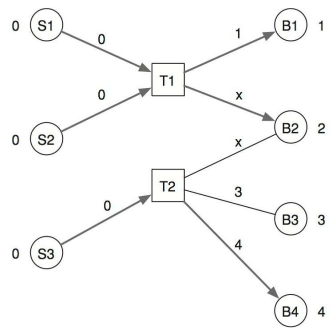

$0 \leq x \leq 2$

Ripple effects

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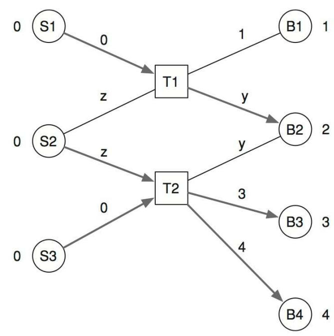

$1 \leq y \leq 2 \qquad 1 \leq z \leq 3$