Actor-Network Procedures (Extended Abstract)

Dusko Pavlovic\textsuperscript{1} and Catherine Meadows\textsuperscript{2}\textsuperscript{*}

\textsuperscript{1} Royal Holloway, University of London, and Universiteit Twente, EWI/DIES, dusko.pavlovic@rhul.ac.uk
\textsuperscript{2} Naval Research Laboratory, Code 5543, Washington, DC 20375 catherine.meadows@nrl.navy.mil

Abstract. In this paper we propose actor-networks as a formal model of computation in heterogeneous networks of computers, humans and their devices, where these new procedures run; and we introduce Procedure Derivation Logic (PDL) as a framework for reasoning about security in actor-networks, as an extension of our previous Protocol Derivation Logic. Both formalisms are geared towards graphic reasoning. We illustrate its workings by analysing a popular form of two-factor authentication.

1 Introduction

Over the last few years, almost without being aware of it, we have been seeing a marked change in our view computation and networking is. We are moving beyond networks of computers communicating on channels constructed out of wires and routers to networks of human beings and various types of devices communicating over multiple channels: wired, wireless, cellular, as well as human usable channels based on voice and vision.

This change has been particularly relevant to security, and to the development and analysis of security protocols. Over the past twenty years or so, there has been extensive, and often influential work on the development of formal methods for the analysis of security protocols. One of the secrets of the success of this work is that it has been based on a simple but powerful model, first introduced in the late thirty years ago by Dolev and Yao [14], in which abstract principals communicate across a network controlled by a hostile intruder. This model has made it possible to develop both model checkers for determining whether or not attacks are possible, and logical systems for determining what a principal can conclude as a result of participating in a protocol. However, this network model is harder to apply in a heterogeneous networks using multiple types of channels.

Our goal is to contribute towards a formal framework for for reliable and practical reasoning about security of computation and communication in network engineering. Towards this goal, we draw our formal models from the informal reasoning practices, and attempt to make them mathematically precise,

\textsuperscript{*} Supported by ONR
while trying to keep them as succinct and intuitive as possible. The main feature of our formalism is that it provides support for diagrammatically based security proofs, which we illustrate by examples in this paper. Our approach is intended to capture what we consider the two most salient concepts for security in this new paradigm for computing: the coalitions that are formed between humans and devices in order to enable secure computation and communication in these emerging networks, which we describe via the use of actor-networks, a concept borrowed from sociology, and the orchestration of different types of communication along different types of channels, which we describe by generalizing the notion of protocol to that of a procedure. These ideas are described in more detail below.

**Actor-networks.** Networks have become an immensely popular model of computation across sciences, from physics and biology, to sociology and computer science [15, 33, 31]. Actor-networks [25] are a particularly influential paradigm in sociology, emphasizing and analyzing the ways in which the interactions between people and objects, as equal factors, drive social processes, in the sense that most people cannot fly without an airplane; but that most airplanes also cannot fly without people. Our goal in the present paper is to formalize and analyze some security processes in networks of people, computers, and the ever-expanding range of devices and objects used for communication and networking, blurring many boundaries. The idea that people, computers, and objects are equal actors in such networks imposed itself on us, through the need for a usable formal model, even before we had heard of the sociological actor-network theory. After we heard of it, we took the liberty of adopting the name actor-network for a crucial component of our mathematical model, since it conveniently captures many relevant ideas. While the originators of actor-network theory never proposed a formal model, we believe that the tasks, methods and logics that we propose are not alien to the spirit of their theory. In fact, we contend that computation and society have pervaded each other to the point where computer science and social sciences already share their subject.

**Procedures.** In computer programs, frequently used sequences of operations are encapsulated into procedures, also called routines. A procedure can be called from any point in the program, and thus supports reuse of code.

In computer networks, frequently used sequences of operations are specified and implemented as network protocols, or as cryptographic protocols. So protocols are, in a sense, network procedures. Beyond computer networks, there are now hybrid networks, where besides computers with their end-to-end links, there may be diverse devices, with their heterogenous communication channels, cellular, short range etc. Online banking and other services are nowadays usually secured by two-factor and multi-factor authentication, combining passwords with smart cards, or cell phones. A vast area of multi-channel and out-of-band protocols opens up, together with the web service choreographies and orchestrations; and we have only scratched its surface. And then there are of course
also social networks, where people congregate with their phones, their cameras and their smiling faces, and overlay the wide spectrum of their social channels over computer networks and hybrid networks. Many sequences of frequently used operations within these mixed communication structures have evolved. This is what we call actor-network procedures.

Outline of the paper. The remainder of the paper is organized as followed. In Sec. 2, we give an overview of related work, in particular the work that has most contributed to our own. Sec. 3 introduces the formal model of actor-networks. Sec 4 explains how actor-networks compute, and introduces the formalisms to represent that computation, all the way to actor-network procedures. Sec. 5 presents Procedure Derivation Logic (PDL) as a method for reasoning about actor-network procedures. In Sec. 6 we provide the first case studies using PDL: we analyze the two-factor authentication in online banking. Sec. 7 contains a discussion of the results and the future work.

2 Related work

In social and computational networks, procedures come in many flavors, and have been studied from many angles. Besides cryptographic protocols, used to secure end-to-end networks, in hybrid networks we increasingly rely on multi-channel protocols [44], including device pairing [24]. In web services, standard procedures come in two flavors: choreographies and orchestrations [41]. There are, of course, also social protocols and social procedures, which were developed and studied first, although not formally modeled. As social networks are increasingly supported by electronic networks, and on the Web, social protocols and cryptographic protocols often blend together. Some researchers have suggested that the notion of protocol should be extended to study such combinations [6, 19, 23]. On the other side, the advent of ubiquitous computing has led to extensive, careful, but largely informal analyses of the problems, e.g., of device pairing, and of security interactions of using multiple channel types [44, 22, 32]. One family of device pairing proposals has been systematically analyzed in the computational model in [45, 34, 26, 27].

There is a substantial and extremely successful body of research on the formal specification and verification of security protocols. These can be roughly be divided into work in the cryptographic model, which directly formalizes cryptographic reasoning, and the symbolic model, which represents data and computations symbolically as terms and operators in a term algebra. The symbolic approaches, in turn, can be divided into two approaches. The first relies on model checking, which is used to implement exhaustive search for attacks. This often comes together with proofs that exploration of a certain finite search space without finding an attack guarantees security. See [4] for a history and survey of model checking security protocols. The second builds on logical systems that are used to derive what a participant in a protocol can conclude after the completion of a run. The earliest logical system for cryptographic protocol analysis was the
Burrows-Abadi-Needham logic [7]. A number of successful tools and logics followed. More recent work that has concentrated applying logical reconstructions includes the Cryptographic Protocol Shape Analyzer (CPSA) [13], the Protocol Composition Logic (described in more detail below), and our own Protocol Derivation Logic, which we use as the basis for logical framework developed in this paper.

It is well understood that model checking, by producing explicit attacks, can be very useful in aiding understanding of a protocol and where it has gone wrong. What seems to be less well appreciated is that the use of logical reconstructions can also give insight, but in a different way, since they give a clearer picture of the assumptions that are necessary for the security of a protocol, as well as the ways in which various pieces of the protocol contribute to its security. When combined with a diagrammatic reasoning, this can give considerable insight into the structure and applicability of a protocol. This is very important, since protocols are often reused and redesigned for different environments, with different security assumptions and types of communication channels. This approach of combining logical reconstruction with diagrammatic reasoning is the one we take in this paper.

Our formal model, as well as the basis for its diagrammatic support, is derived from the strand space model [20]. Among its many salient features, the convenient diagrammatic protocol descriptions of strand spaces has been an important reason for their wide acceptance and popularity. It is important to note that the strand space diagrams are not just an intuitive illustration, but that they are formal objects, corresponding to precisely defined components of the theory, while on the other hand closely resembling the informal “arrows-and-messages” protocol depictions, found in almost every research paper and on almost every white board where a protocol is discussed.

Protocol Composition Logic (PCL) was, at least in its early versions [18, 12, 10, 17, 11], an attempt to enrich the strand model with a variable binding and scoping mechanism, making it into a process calculus with a formal handle on data flows, which would thus allow attaching Floyd-Hoare-style annotations to protocol executions, along the lines of [39, 40]. This was necessary for incremental refinement of protocol specifications, and for truly compositional, and thus scalable protocol analyses, which were the ultimate goal of the project. However, less attention was paid to the purely diagrammatic contribution of the strand space model.

Protocol Derivation Logic (PDL) has been an ongoing effort [29, 8, 36, 2, 30, 37] towards a scalable, i.e. incremental protocol formalism, allowing composition and refinement like PCL, but equipped with an intuitive and succinct diagrammatic notation, like strand spaces. The belief that these two requirements can be reconciled is based on the observation that the reasoning of protocol participants is concerned mostly with the order of events in protocol executions. It follows that the protocol executions and their logical annotations both actually describe the same structures, which can be viewed as partially ordered multisets [43], and manipulated within the same diagrammatic language. This has been the guiding
idea of PDL. Several case studies of standard protocols, and the taxonomies of the corresponding protocol suites, have been presented in [29, 8, 36, 2]. An application to a family of distance bounding protocols has been presented in [30]; and an extension supporting the probabilistic reasoning necessary for another such family has been proposed in [37]. In the present paper, we propose the broadest view of PDL so far — which should here be read as Procedure Derivation Logic. Since cryptographic protocols are usually construed as the tools of computer security, we use the term procedure here to denote a frequently used pattern of operations in a modern network, which may include computers and software agents, but also humans, as well as various kinds of communication devices. Procedure Derivation Logic is thus our attempt to address the need for formal reasoning about the pervasive security problems, that arise at the interfaces of cyber space with physical and social spaces, as discussed in [38]. It should be mentioned that the mosaic of protocol logics, which we are thus attempting to expand beyond protocols, is, in a certain sense, counterbalanced by the more homogenous (if not entirely monolithic) world of computational modeling. Following the seminal work in [1], it has become customary to verify that every symbolic model, underlying a protocol logic, is sound when interpreted computationally. Such interpretations have led to many interesting results and useful tools [5, 3, 9]. It should be clear, however, that the standard computational model does not allow modeling many of the physical or social features that we are trying to capture in procedures. Since our execution model includes network nodes that represent, e.g., humans, and the visual channels through which these humans can look at each other, it does not seem reasonable to try to interpret it computationally. And a model can, of course, only be computationally sound, or unsound, relative to a specific computational interpretation. Without such an interpretation, the question of computational soundness cannot be stated.

3 Actor-network model

We model network computation in terms of (1) computational agents, some of them controlled by various parties, others available as resources, and (2) communication channels between the agents, supporting different types of information flows.

3.1 Formalizing actor-networks

Definition 3.1 An actor-network consists of the following sets: (1) identities, or principals $\mathcal{J} = \{A, B, \ldots\}$, (2) nodes $\mathcal{N} = \{M, N, \ldots\}$, (3) configurations $\mathcal{P} = \{P, Q, \ldots\}$, where a configuration can be a finite set of nodes, or a finite set of configurations; (4) channels $\mathcal{C} = \{f, g, \ldots\}$, and (5) channel types $\Theta = \{\tau, \varsigma, \ldots\}$ given with the structure $\Theta \leftarrow \mapsto \mathcal{C} \Rightarrow \mathcal{P} \overset{\odot}{\Rightarrow} \mathcal{J}$ where the partial map $\overset{\odot}{\Rightarrow} : \mathcal{P} \to \mathcal{J}$ tells which principals control which configurations, the pair of maps...
\[ \delta, \varrho : C \to P \text{ assign to each channel } f \text{ an entry } \delta f \text{ and an exit } \varrho f, \text{ and the map } \vartheta : C \to \Theta \text{ assigns to each channel a type.} \]

An actor is an element of a configuration.

Notation. We denote by \( N_B \) a node \( N \) controlled by the principal \( \circ N = B \). We write \( g = (P \to N_B) \) for a channel \( g \) of type \( \vartheta g = \tau \), with the entry \( \delta g = P \), and with the exit \( \varrho g = N \) controlled by \( \circ N = B \). Since there is usually at most one channel of a given type between two given configurations, we usually omit the label \( g \), and write just \( P \to N_B \) to denote this channel.

3.2 Example: an actor-network for two factor authentication

To mitigate phishing attacks, some online banks have rolled out two factor authentication. This means that they do not just verify that the user knows a password, but also something else — which is the second authentication factor. This second factor often requires some additional network resources, besides the internet link between the customer and the bank. This is the first, quite familiar step beyond simple cyber networks.

Some banks authenticate that the user is in possession of her smart card. The underlying actor-network is on Fig. 1. The user Alice controls her computer \( C_A \) and her smart card \( S_A \). She is also given a portable smart card reader \( R \). She inserts the card in the reader to form the configuration \( Q \). The reader is available to Alice, but any other reader would do as well. Configured into \( Q \), the smart card and the reader verify that Alice knows the PIN, and then generates the login credentials, which Alice copies from \( R \)'s screen to her computer \( C_A \)'s keyboard, which forwards it to bank Bob’s computer \( C_B \). The details of the authentication procedure will be analyzed later.

In summary, the network thus consists of principals \( J = \{A, B\}, N = \{I_A, C_A, S_A, R, C_B\} \); configurations \( P = N \cup \{Q\} \), where \( Q = \{S_A, R\} \), and the following six channels: (1 and 2) cyber channels \( C_A \rightleftharpoons C_B \) between Alice’s and Bank’s computers, (3) a visual channel \( C_A \to I_A \) from Alice’s computer to her human \( I_A \), (4) a keyboard \( I_A \to C_A \) from Alice’s human to her computer, (5) a visual channel \( R \to I_A \) from the smart card reader to Alice’s human, and (6) a keyboard \( I_A \to R \) from Alice’s human to the card reader.

![Fig. 1. A pervasive network: Online banking with a smart card reader](attachment:image.png)
4 Actor-network processes

4.1 Computation and communication

Computation in a network consists of events, which are localized at nodes or configurations. An event that is controlled by a principal is an action.

Communication in a network consists of information flows along the channels. Each flow corresponds to a pair of events: (1) a write event at the entry of the channel, and (2) a read event at the exit of the channel. There are two kinds of flows: (1) messages, which consist of a send action at the entry of the channel, and a receive coaction at the exit, and (2) sources, which consist of an sample action at the exit, and a emit coaction at the entry.

Besides transferring information from one configuration to another, the flows also synchronize the events that take place at different localities, because: every receive coaction must be preceded by a corresponding send action, and every sample action must be preceded by a corresponding emit coaction.

If a source has not been emitted to anywhere, then there is nothing to sample, and no sampling of that source can occur. If a message has not been sent, then the corresponding receive event cannot occur. So when I receive a message, then I know that it must have been sent previously by someone; and when I sample a source, then I know that someone must have emitted to this source. That is how I draw conclusions about non-local events from the observations of my own local actions. This is formalized in Sec. 5.2.

4.2 Formalizing data as terms

Each flow carries some data, which contain information. As is standard in the symbolic protocol model, we represent this as terms in an algebra. Recall that an algebraic theory is a pair \((O, E)\), where \(O\) is a set of finitary operations (given as symbols with arities), and \(E\) a set of well-formed equations (i.e. where each operation has a correct number of arguments) [21].

**Definition 4.1** An algebraic theory \(\mathcal{T} = (O, E)\) is called a data theory if \(O\) includes a binary pairing \((-,-)\) operation, and the unary operations \(\pi_1\) and \(\pi_2\) such that \(E\) contains the equations \(\pi_1(u, v) = u\), \(\pi_2(u, v) = v\), and \(((x, y), z) = (x, (y, z))\). A data algebra is a polynomial extension \(\mathcal{T} [\mathcal{X}]\) of a \(\mathcal{T}\)-algebra \(\mathcal{T}\).

*Function notation.* When no confusion seems likely, we elide the function applications to concatenation, and write \(f.x\) instead of \(f(x)\). When no confusion is likely, we even elide the dot from the concatenation and simply write \(fx\) instead of \(f.x\), or \(f(x)\).

*Random values are represented by indeterminates.* A polynomial extension \(\mathcal{T} [\mathcal{X}]\) is the free \(\mathcal{T}\)-algebra generated by adjoining a set of indeterminates \(\mathcal{X}\) to a \(\mathcal{T}\)-algebra \(\mathcal{T}\) [21, §8]. The elements \(x, y, z \ldots\) of \(\mathcal{X}\) are used to represent nonces and other randomly generated values.

*Easy subterms.* We assume that every data algebra comes equipped with the easy subterm relation \(\sqsubseteq\). The idea is that that \(s \sqsubseteq t\) implies that \(s\) is a subterm
of $t$ such that every principal who knows $t$ also knows $s$. In other words, the views $\Gamma_A$ are lower closed under $\sqsubseteq$, as explained in [36]. This is in contrast with hard subterms, which cannot be extracted: e.g., the plaintext $m$ and the key $k$ are hard subterms of the encryption $E.k.m$.

4.3 Formalizing events and processes

In this section we define processes, the events that processes engage in, and the ordering of events within a process.

An event or action is generally written in the form $a[t]$ where $a$ is the event identifier, and $t$ is the term on which the event may depend. When an event does not depend on data, the term $t$ is taken to be a fixed constant $t = \sqrt{e}$, and we often abbreviate $a[\sqrt{e}]$ to $a$.

The most important events for our analyses are the action-coaction couples send-receive, and sample-emit, for which we introduce special notations: send \(\langle \cdot t \rangle\), receive \((\cdot t\rangle\), and emit \(\langle t : \rangle\), sample \((t:)\). Generically, we write \(\langle t \rangle\) for a write action, which can be either \(\langle \cdot t \rangle\) or \(\langle t : \rangle\), and \((t:)\) for a read action, which can be either \((\cdot t\rangle\) or \((t:)\). Another often used action is $\nu[x]$ for the generation of a random value. It could also be implemented as sampling a source of randomness represented as a devoted node. In addition, the nodes are capable of performing various local operations, which are specified in the definition of the procedure.

For actions, such as \(\langle \cdot t \rangle\) and \((t:)\), the configuration $P$ must be controlled, i.e. the partial function $\circ : N \rightarrow J$ must have a definite value $\circ P$.

**Definition 4.2** A process $F$ is a partially ordered multiset of localized events, i.e. a mapping

$$F = (F_\mathcal{E}, F_P) : \mathcal{E} \rightarrow \mathcal{E} \times \mathcal{P}$$

where $(\mathcal{E}, \rightarrow)$ is a well-founded partial order, representing the structure time, $\mathcal{E}$ is a family of events, and $(\mathcal{P}, \subseteq)$ the partial order of configurations, and they satisfy the requirements that

(a) if $F_\mathcal{E} \phi$ is an action, then $\circ (F_P \phi)$ is well defined, and
(b) if $\phi \rightarrow \psi$ in $\mathcal{E}$ then $F_P \phi \subseteq F_P \psi$ or $F_P \phi \supseteq F_P \psi$ in $\mathcal{P}$.

**Notation:** The points in time are denoted by events. By abuse of notation, we usually write $a[t\rangle_P$ for $\phi \in F$ where $F_\mathcal{E} \phi = a[t]$ and $F_P = P$.

(a) if an action takes place at a configuration $P$, then $P$ is controlled, i.e. $\circ P$ must be well defined, and
(b) if $a[t\rangle_P \rightarrow b[s\rangle_Q$ then $P \subseteq Q$ or $P \supseteq Q$.

**Definition 4.3** We say that the term $t$ originates at the point $\phi \in F$ if $\phi$ is the earliest write of a term containing $t$. Formally, $\phi$ thus satisfies $F_\mathcal{E} \phi = (s)$ where $t \subseteq s$, and $F_\mathcal{E} \xi = (s) \land t \subseteq s \Rightarrow \phi \rightarrow \xi$ holds for all events $\xi$.

**Notation:** *Origination*. We extend the notational conventions described above by denoting by $\sqrt{\langle t \rangle}_P$ the event $\phi$ where the term $t$ originates. The configuration $P$ is the originator of $t$. 


4.4 Formalizing flows, runs and procedures

We now extend our discussion to the definition of communication between processes, and extend our ordering to events occurring within a procedure as well as individual processes.

We begin by defining a more general version of channel between two configurations, called a flow channel. A flow channel exists between any two configurations if a channel exists between any two nodes on the configuration trees. It is called a flow channel because the information passed along the channel flows upwards to the configuration as a whole. It is defined formally below.

**Definition 4.4** For configurations $P, Q \in \mathcal{P}$, a flow channel $P \stackrel{\tau}{\rightarrow} Q$ can be either (1) a channel $P \rightarrow Q$, (2) a flow channel $P \stackrel{\tau}{\rightarrow} Q'$, where $Q' \in Q$, (3) a flow channel $P' \stackrel{\tau}{\rightarrow} Q$, where $P' \in P$, or (4) a flow channel $P' \stackrel{\tau}{\rightarrow} Q'$, where $P' \in P$ and $Q' \in Q$.

A flow $a[t]_P \stackrel{\tau}{\rightarrow} b[s]_Q$ is given by a flow channel $P \stackrel{\tau}{\rightarrow} Q$, and an interaction pair $a[t], b[s]$, i.e. a pair where either $a[t] = \langle t \cdot \rangle$ and $b[s] = \langle s \cdot \rangle$, or $a[t] = \langle t ; \rangle$, and if $b[s] = \langle s ; \rangle$.

A flow $a[t]_P \stackrel{\tau}{\rightarrow} b[s]_Q$ is complete if $s = t$.

**Definition 4.5** Let $\mathcal{F}$ be a process. A run, or execution $\mathcal{E}^\mathcal{F}$ of $\mathcal{F}$ is an assignment for each coaction $b[s]_Q$ of a unique flow $a[t]_P \rightarrow b[s]_Q$, which is required to be sound, in the sense that $b[s]_Q \neq a[t]_P$ in $\mathcal{F}$.

A run is complete if all of the flows that it assigns are complete: the terms that are received are just those that were sent, and the inspections find just those terms that were submitted.

A run is a pomset extending its process. Setting $a[t]_P \rightarrow b[s]_Q$ whenever there is a flow $a[t]_P \rightarrow b[s]_Q$ of some type $\tau$ makes a run $\mathcal{E}^\mathcal{F}$ into an extension of the ordering of the process $\mathcal{F}$, as a partially ordered multiset. The pomset $\mathcal{E}^\mathcal{F}$ does not have to satisfy condition (b) of Def. 4.2 any more. Indeed, the whole point of running a process is to extend in $\mathcal{E}^\mathcal{F}$ the internal synchronizations, given by the ordering of $\mathcal{F}$, with the additional external synchronizations.

**Definition 4.6** A network procedure $\mathcal{L}$ is a pair $\mathcal{L} = \langle \mathcal{F}_\mathcal{L}, \mathcal{E}_\mathcal{L} \rangle$ where $\mathcal{F}_\mathcal{L}$ is a process, and $\mathcal{E}_\mathcal{L} = \{ \mathcal{E}_1^\mathcal{F}_\mathcal{L}, \mathcal{E}_2^\mathcal{F}_\mathcal{L}, \mathcal{E}_3^\mathcal{F}_\mathcal{L}, \ldots \}$ is a set of runs of $\mathcal{F}_\mathcal{L}$. The elements of $\mathcal{E}_\mathcal{L}$ are called secure runs. All other runs are insecure. A procedure is said to be secure if every insecure run can be detected by a given logical derivation from the observations of a specified set of participants.

**Graphic presentations of procedures.** To specify a procedure $\mathcal{L}$, we draw a picture of the pomset $\mathcal{F} = \mathcal{F}_\mathcal{L}$, and then each of its extensions $\mathcal{E} = \mathcal{E}_i^\mathcal{F}$. Because of condition (b) of Def. 4.2, the events comparable within the ordering of a process $\mathcal{F}$ must happen within a maximal configuration. Therefore, if the diagram of the partially ordered multiset $\mathcal{F}$ is drawn together with the underlying network, then each component of the comparable events can all be depicted under the corresponding configuration. We can thus draw the network above the process,
and place the events occurring at each configuration along the imaginary vertical lines flowing, say, downwards from it, like in Fig. 3. The additional ordering, imposed when in a run $E$ the messages get sent and the facts get observed, usually run across, from configuration to configuration. This ordering can thus be drawn along the imaginary horizontal lines between the events, or parallel with the channels of the network. Such message flows can also be seen in Fig. 3. The dashed lines represent the data sharing within a configuration.

4.5 Examples of procedures

![Diagram](image)

Fig. 2. Challenge-Response (CR) protocol template

**Challenge response authentication protocols.** We begin a familiar special case of a procedure: a protocol. A large family of challenge-response authentication protocols is subsumed under the template depicted on Fig. 2. Bob wants to make sure that Alice is online. It is assumed that Alice and Bob share a secret $k_{AB}$, which allows them to define functions $c_{AB}$ and $r_{AB}$ such that

- $r_{AB}x$ can be computed from $c_{AB}x$ using $s_{AB}$, but
- $r_{AB}x$ cannot be computed from $c_{AB}x$ alone, without $s_{AB}$.

So Bob generates a fresh value $x$, sends the challenge $c_{AB}x$, and if he receives the response $r_{AB}x$ back, he knows that Alice must have been online, because she must have originated the response. The idea behind this template has been discussed, e.g., in [29, 8, 36, 37]. The template instantiates the concrete protocol components by refining the abstract functions $c_{AB}$ and $r_{AB}$ to concrete implementations, which satisfy the above requirements: e.g., $c_{AB}$ may be the encryption by Alice’s public key, and $r_{AB}$ may be the encryption by Bob’s public key, perhaps with Alice’s identity.
Two-factor authentication procedure. Next we describe the first nontrivial procedure, over the actor-network described in Sec. 3.2. It can be viewed as an extension of the simple challenge-response authentication. There, Bob authenticates Alice using her knowledge of a secret $s^{AB}$, which they both know. Here Bob authenticates that that knows a secret $p^A$ that Bob does not know, and that she has a security token $S_A$, in this case a smart card. The secret and the smart card are the “two factors”. This is the idea of the procedure standardized under the name Chip Authentication Programme (CAP), analyzed in [16]. The desired run of the challenge-response option of this procedure is depicted on Fig. 3.

We assume that, prior to the displayed run, Alice the customer identified herself to Bob the bank, and requested to be authenticated. Bob’s computer $C_B$ then extracts a secret $s^{AB}$ that he shares with Alice. This time, though, the shared secret is too long for Alice’s human $I_A$ to memorize, so it is is stored in the smart card $S_A$. Just like in CR protocol above, Bob issues a challenge, such that the response can only be formed using the secret. So Bob in fact authenticates the smart card $S_A$. He entrusts the smart card $S_A$ with authenticating Alice’s human $I_A$. This is done using the secret $p^A$ shared by $I_A$ and $S_A$. The secret is stored in both nodes. To form the response to Bob’s challenge, Alice forms the configuration $Q$ by inserting her card $S_A$ into the reader $R$. The configuration $Q$ requests that $I_A$ enters the secret PIN (Personal Identification Number) $p^A$ before it forms the response for Bob. There is no challenge from $Q$ to $I_A$, and thus no freshness guarantees in this authentication: anyone who sees $I_A$’s response can replay it at any time. Indeed, the human $I_A$ cannot be expected to perform computations to authenticate herself: most of us have trouble even submitting just the static PIN. The solution is thus to have the card-reader configuration $Q$ computes the response, which Alice relays it to Bob. The old PIN authentication is left to just convince $Q$ that Alice’s human $I_A$ is there: $Q$ tests $p^A$, sent through the keyboard channel from $I_A$ to the reader $R$, coincides with $p^A$ stored in the card $S_A$, and then generates a keyed hash $H_{s^{AB}}x$ using the shared secret $s^{AB}$ and the challenge $x$. This hash is displayed for Alice on the card reader $R$ as the response $r$, which Alice then sends to her computer $C_A$ by the keyboard channel, and further to $C_B$ by the cyber channel.

5 Procedure Derivation Logic

5.1 The language of PDL

A statement of PDL is in the form $A : \Phi$, where $A \in \mathcal{J}$ is a principal, and $\Phi$ is a predicate asserted by $A$. The predicate $\Phi$ is formed by applying logical connectives to the atomic predicates, which can be (1) $a[t]_P$ — meaning “the event $a[t]_P$ happened”, or (2) $a[t]_P \rightarrow b[s]_Q$ — meaning “the event $a[t]_P$ happened before $b[s]_Q$.”

5.2 Communication axioms

The statements of PDL describe the events that happen in a run of a process, and their order. The basic PDL statements are its axioms, which we describe
next. They are taken to be valid in all runs of all processes. The other valid statements are derived from them.

**Origination.** The origination axioms say that any message that is received must have been sent, and that any source that is sampled must have been emitted to. This has been explained early in Sec.4. More precisely, any principal that controls a configuration $P$ where a message is received knows that it must have been sent by someone, no later than it was received; and similarly for a source that is sampled. Formally

$$\circ \mathcal{P} : \langle \cdot t \cdot \rangle_P \implies \exists X. \langle \cdot t \cdot X \rangle_X \rightarrow \langle \cdot t \cdot \rangle_P \quad \text{(orig,m)}$$

$$\circ \mathcal{P} : \langle \cdot t \cdot \rangle_P \implies \exists X. \langle \cdot t \cdot X \rangle_X \rightarrow \langle \cdot t \cdot \rangle_P \quad \text{(orig,s)}$$

**Freshness.** In Sec. 4.2 we explained the idea of modeling random values as the indeterminates in polynomial algebras of messages. The freshness axiom extends this idea to processes, by requiring that each indeterminate $x$ must be (1) *freshly generated* by an action $\nu[x]$ before it is used anywhere, and (2) that it can only be used elsewhere after it has passed in a message or a source, which formally
becomes
\[
\text{fresh.1)} \quad \circ P : a[t.x]_P \Rightarrow \exists X. \, \nu[x]_X \Rightarrow a[t.x]_P
\]
\[
\text{fresh.2)} \quad \circ P : \neg \nu[x]_P \land a[t.x]_P \Rightarrow \exists X. \, (\nu[x]_X \Rightarrow (\langle : x :: \rangle)_X \Rightarrow (\langle : x :: \rangle)_P \Rightarrow a[t.x]_P)
\]

where, using the easy subterm order \( \sqsubseteq \) from Sec. 4.2, \( \langle : x :: \rangle_X \) abbreviates \( \exists t. \, x \sqsubseteq t \land (\langle : t :: \rangle)_X \), \( \langle : x :: \rangle_X \) abbreviates \( \exists t. \, x \sqsubseteq t \land (\langle : t :: \rangle)_X \), etc.

### 5.3 Authentication axioms

In our model, there are two forms of authentication: interactions along authentic channels, and challenge-response authentication.

**Interactions along authentic channels.** An authentic channel allows at least one of the participants to observe not only the events on their own end of the channel, but also on the other end. So there are four types of authentic channels, supporting the following assertions:

\[
\text{fresh.1)} \quad \circ P : \langle : t :: \rangle_P \Rightarrow (\langle : t :: \rangle)_Q \quad \text{(auch.m.1)}
\]
\[
\text{fresh.2)} \quad \circ Q : \langle : t :: \rangle_P \Rightarrow (\langle : t :: \rangle)_Q \quad \text{(auch.m.2)}
\]

Channels that satisfy \text{auch.m.1} or \text{auch.p.1} are called \text{write}-authentic; channels that satisfy \text{auch.m.2} or \text{auch.p.2} are called \text{read}-authentic. Here are some examples from each family:

- \text{(auch.m.1):} A keyboard channel guarantees to the sender that the device at which she is typing is receiving the message
- \text{(auch.m.2):} A visual channel used for sending a message allows the receiver to see the sender.
- \text{(auch.p.1):} When my fingerprints are taken, I observe that they are taken, and can see who is taking them.
- \text{(auch.p.2):} Moreover, the person taking my fingerprints also observes that they are taking my fingerprints.

Besides these assertions about the order of events, some authentic channels support other assertions. They are usually application specific, and we impose them as procedure specific axioms.

**Challenge-response authentication.** The challenge-response axiom is in the form
\[
\text{cr)} \quad \circ P : \text{Local}_P \Rightarrow \text{Global}_{PQ}
\]
where, using the notation from Sec. 5.2

\[ \text{Local}_P = \nu[x]_P \to \left\langle \cdot c^{PQ}x \cdot \right\rangle_p \to \left\langle \cdot r^{PQ}x \cdot \right\rangle_p \]

\[ \text{Global}_{PQ} = \nu[x]_P \to \left\langle \cdot c^{PQ}x \cdot \right\rangle_p \to \left( \left\langle \cdot c^{PQ}x \cdot \right\rangle_Q \right) \to \sqrt{\left\langle \cdot r^{PQ}x \cdot \right\rangle_Q} \to \left\langle \cdot r^{PQ}x \cdot \right\rangle_p \]

Translated into words, \((cr)\) says that the owner \(\mathcal{O}P\) of the configuration \(P\) knows that (1) if he generates a fresh \(x\), sends the challenge \(c^{PQ}x\), and receives the response \(r^{PQ}x\), then (2) \(Q\) must have received a message containing \(c^{PQ}x\) after he sent it, and then she must have sent a message containing \(r^{PQ}x\) before he received it.

Using \((cr)\) and certain observations of the local events at \(P\), the principal \(\mathcal{O}P\) can thus draw the conclusions about certain non-local events at \(Q\), which he cannot directly observe. Fig. 4 depicts this reasoning diagrammatically.

**Remark.** The \((cr)\) axiom, and the corresponding protocol template, displayed on Fig. 5, has been one of the crucial tools of the Protocol Derivation Logic, all the way since [29, 8], through to [37].

6 Examples of reasoning in PDL

6.1 On the diagrammatic method

In its diagrammatic form depicted on Fig. 5, axiom \((cr)\) says that the verifier \(P\), observing the local path on the left, can derive the path around the non-local actions on the right. This pattern of reasoning resembles the categorical practice of *diagram chasing* [28, 35]. Categorical diagrams are succinct encodings of
lengthy sequences of equations. Just like the two sides of the implication in (cr) correspond to two paths around Fig. 5, the two sides of an equation are represented in a categorical diagram as two paths around a face of that diagram. The components of the terms in the equations correspond to the individual arrows in the paths. The equations can be formally reconstructed from the diagrams. Moreover, the diagrams can be formally combined into new proofs. The algebraic structures are thus formally transformed into geometric patterns. After some practice, the geometric intuitions begin to guide algebraic constructions in the formal language of diagrams. We apply a similar strategy to PDL.

6.2 Cryptographic (single-factor) authentication

We begin with a simple example of diagrammatic reasoning, present already in [29].

**Theorem 6.1** The functions $c^{PQ}x = x$ and $r^{PQ}x = \varsigma^Qx$ implement (cr), provided that the abstract signature function $\varsigma$ satisfies the following axioms:

(a) $\varsigma^Qu = \varsigma^Qv \implies u = v$, i.e., $\varsigma^Q$ is injective,
(b) $\sqrt{\langle\langle \varsigma^Q t \rangle\rangle}_X \implies X = Q$, i.e., $\varsigma^Q t$ must originate from $Q$,
(c) $V^Q u \cdot t \iff u = \varsigma^Q t$, i.e., the predicate $V^Q$ is satisfied just for the pairs $u, t$ where $u = \varsigma^Q t$,

and that these axioms are known to the principal $\text{Bob} = \odot P$. 

**Proof.** To prove the claim, we chase the diagram on Fig. 10. The numbered arrows arise from the following steps:

1. Bob = $\odot P$ observes $\nu[x]_P \to \langle \cdot x \cdot \rangle_ P \to \langle \cdot r [V^Q \cdot x \cdot] \rangle_ P$, i.e. after sending a fresh value $x$, he receives a response $u$ which passes the verification $V^Q x$.
2. Using the axioms (c) and (orig.m), he concludes that there is some $X$ such that $\langle \cdot V^Q x \cdot \rangle_X \to \langle \cdot r [V^Q \cdot x \cdot] \rangle_ P$.
3. Using (fresh.2) he further derives that for the same $X$ holds $\langle \cdot x \cdot \rangle_X \to (\langle \cdot x \cdot \rangle_X) \to \langle \cdot V^Q x \cdot \rangle_X$.
4. Using (a) and (b), Bob concludes that $V^Q x$ must have originated from $Q$.

\[ \square \]

6.3 Pervasive (two-factor) authentication

Next we describe how Bob the bank authenticates Alice the customer in the CAP procedure.

**Theorem 6.2** The procedure on Fig. 3 implements authentication, i.e. satisfies (cr), provided that the following assumptions are true, and known to Bob:

(a) $Hu = Hv \implies u = v$, i.e., $H$ is injective;
Proof. Prior to the displayed execution, Alice is assumed to have sent to Bob her identity, and a request to be authenticated. Following this request, Bob’s computer $C_B$ has extracted the secret $s^{AB}$ from a store, which he will use to verify that $S_A$ has generated the response.

To prove the claim, we chase the diagram on Fig. 6. The enumerated steps in the diagram chase correspond to the following steps in Bob’s reasoning:

1. Bob observes $\nu[x]_{C_B} \rightarrow \langle \cdot x \cdot \rangle_{C_B} \rightarrow \langle \cdot Hs^{AB}x \cdot \rangle_{C_B}$.
2. Using $(\text{orig-m})$ he concludes that there is some $X$ such that $\langle \cdot Hs^{AB}x \cdot \rangle_X \rightarrow \langle \cdot Hs^{AB}x \cdot \rangle_{C_B}$.
3. Using $(\text{fresh.}2)$ he further derives that for the same $X$ holds $\langle \cdot x \cdot \rangle_{C_B} \rightarrow \langle \cdot x \cdot \rangle_X \rightarrow \langle \cdot Hs^{AB}x \cdot \rangle_X$.
4. By (a) and (b), from the observation that he did not use $s^{AB}$, Bob concludes that $Hs^{AB}$ must have originated in a configuration $Q$ containing $S_A$.
5. By (c), $\langle \cdot p^A \cdot \rangle_{I_A} \rightarrow \langle \cdot p^A \cdot \rangle_Q \rightarrow \langle p^A = \overline{p} \rangle \rightarrow \langle Hs^{AB}x \rangle$, where the last action abbreviates $(r := Hs^{AB}x)$, and we write out $r$ as $Hs^{AB}x$ in the rest of the diagram.
6. Since $Q$ had to also receive $x$ before computing the response in $\langle \cdot p^A, x \cdot \rangle_{R} \rightarrow (Hs^{AB}x)$ follows by (d). So $\langle \cdot p^A, x \cdot \rangle_{Q}$ from 5 is $\langle \cdot p^A, x \cdot \rangle_{R}$.
7. By (\text{orig-m}), there is $Y$ with $\langle \cdot p^A, x \cdot \rangle_{Y} \rightarrow \langle \cdot p^A, x \cdot \rangle_{R}$. By (e), $\langle \cdot p^A \cdot \rangle_{I_A}$ from 5 must be $\langle \cdot p^A, x \cdot \rangle_{I_A}$.
8. The fresh value $x$ has thus been sent to $Q$ by $I_A$. It follows that in 2 and 3 above must be $X = I_A$.
9. Since $A$ controls $S_A$ and $I_A$, and $S_A \in Q$ generated the response $Hs^{AB}x$, only $I_A$ could have sampled $Hs^{AB}x$ along the visual channel.
10. Since $A$ controls $I_A$ and $C_A$, only $I_A$ could have sent $Hs^{AB}x$ to $C_A$ along the keyboard channel.

These logical steps suffice to assure Bob that if he observes the local flow on the right in Fig. 6, then the non-local flow along the external boundary, all the way to the left side of the diagram and back must have taken place. Comparing this diagrammatic conclusion with the pattern of $(\text{cr})$ on Fig. 5, we see that Bob has proven an instance of authentication.

We have provided a security proof of the CAP protocol discussed in [16]. However, the discussion in that paper is devoted to pointing out the security risks inherent in relying upon that very protocol. How can this happen? As it turns out, we can reproduce the situations that the authors in [16] warn against by relaxing the assumptions that our proof relies upon. This is one advantage of combining logical reconstruction with explicit specifications of the configurations and channels involved.
Fig. 6. B’s reasoning in CAP
Relaxing the assumption that Alice is honest: In this case $\sqrt{\langle(p^A)\rangle_X}$ fails to hold, because if Alice is not honest she could turn PIN over to a third party. This is discussed in [16] in terms of the card being stolen and Alice being intimidated into revealing her PIN.

Relaxing the assumption that $R$ is honest: in this case $\sqrt{\langle(p^A)\rangle_X}$ fails to hold again, because $S_A$ or $R$ could reveal the PIN to a third party. This is discussed in [16] in two places. First, the reader could inadvertently reveal the PIN because the keys used to enter it become visibly worn. Secondly, for practical reasons users are often required to use untrusted readers provided by third parties, which could steal the PIN.

Relaxing the assumption that Alice controls $C_A$: if $C_A$ has been infiltrated by malware then Alice no longer controls it. As the reader can verify, this has absolutely no effect on the proof of security of the CAP and PIN protocol in isolation. Indeed, $C_A$ could be replaced by a cyber channel where ever it is used without affecting the protocol’s security. The problem is when the hash computed by the smart card is used to authenticate a bank transaction. The most straightforward way of doing this is for the hash to be passed to $C_A$, which computes, for example, a Message Authentication Code on the transaction. If $C_A$ is not controlled by Alice, it could substitute a different transaction.

7 Conclusion

We have presented a logical framework for reasoning about security of protocols that make use of a heterogeneous mixture of humans, devices, and channels. We have shown how different properties of channels and configurations can be expressed and reasoned about within this framework. A key feature of this framework is that it supports explicit reasoning about both the structure of a protocol and the contributions made by its various components, using a combination of diagrammatic and logical methods. Because of this, we believe that our approach can be particularly useful in giving a more rigorous foundation for white-board discussions, in which protocols are usually displayed graphically. By annotating the diagram with the proof using the methods described in this paper, formal reasoning could be brought to bear at the very earliest stages of the design process.

Acknowledgement. The first author would like to thank Wolter Pieters [42] for introducing him to Bruno Latour’s ideas about actor-networks.

References


